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# Frequency Domain Approach for Transonic Aerodynamic Modelling applied to a Viscous Aircraft Model

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**This work introduces a method for the construction of a reduced order model in the frequency domain. With input data obtained with the TAU linearized frequency domain solver, the reduced order model shows a strong ability to reconstruct the full order frequency response of a viscous aircraft configuration. Reduced order models have been successfully created for different strips on the wing, the fuselage and the horizontal plane.**

## Nomenclature

|            |   |                                   |
|------------|---|-----------------------------------|
| $\alpha$   | = | angle of attack                   |
| $\alpha_0$ | = | amplitude of the pitching motion  |
| $\alpha_m$ | = | mean angle of the pitching motion |
| $C_p$      | = | pressure coefficient              |
| $C_L$      | = | lift coefficient                  |
| $C_D$      | = | Drag coefficient                  |
| $C_M$      | = | Pitching moment coefficient       |
| $F_z$      | = | Vertical force                    |
| $M_Y$      | = | Pitching moment                   |
| $k$        | = | Reduced frequency                 |
| $U_\infty$ | = | Freestream velocity               |

## I. Introduction

Computational Fluid Dynamics (CFD) now has a wide range of validity where it gives highly accurate results compared to wind tunnel experiments. It is extensively used in industry for steady analysis such as performance studies. However, unsteady aerodynamics is also required for aircraft design and aeroelastic applications such as flutter speed or limit cycle oscillation prediction. Whilst more powerful computers have enabled the application of CFD for unsteady loads calculations, in practice the computational cost remains too high for routine use, especially when it comes to viscous flows.

Reduced order models (ROMs) can be constructed that aim to decrease the CPU time for unsteady loads calculations by capturing the dominant behaviour of the numerical model with a small number of degrees of freedom, whilst

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retaining good accuracy and stability. These ROMs enable [1] the study of the system, and establishing control laws to be simplified. Model order reduction can be achieved using different methods; these depend on the physics of the system, the accuracy wanted and the information availability. The latest can be based on physical equations, engineering problems, datasets and so on. For systems whose model is strongly linked to the physics, order reduction can even be performed by hand, thinking about the independencies between the parameters; interpolation can also be used.

While building a ROM, one technique is to define projection bases and spaces. The idea is to use linear algebra and to construct a subspace orthogonal to the Krylov subspace; this can be performed thanks to the Gram-Schmidt orthonormalization method. Since it can be unstable [2] a modified Gram-Schmidt can be used. In order to achieve this, Arnoldi developed an iterative algorithm [3]. If the system matrix is Hermitian, the Lanczos method [4] is much faster. It is based on the Arnoldi method, but as the system matrix is symmetric, the algorithm is much simpler and the recurrence is shorter: each vector  $U_{j+1}$  is directly calculated from the two previous ones  $U_j$  and  $U_{j-1}$ . The Lanczos algorithm can also be combined with a Padé approximation, for a method called Padé via Lanczos (PVL). This method aims to preserve the stability of the system. In fact, the reduced order modeling techniques using the Padé approximation do not ensure this stability [5]. Other methods such as partial PVL [6] enable the poles and the zeros of the reduced transfer function to be corrected; it leads to an enhanced stability. Antoulas [7] uses the advantages of both Krylov subspaces and balanced truncation approaches. Finally, the Passive Reduced-order Interconnect Macromodelling Algorithm, while using the Arnoldi method guarantees the preservation of passivity and enables an enhanced accuracy [8].

An alternative analysis approach uses the system response due to different excitations to identify reduced matrices. Several algorithms have been developed for model reduction using singular value decomposition (SVD) of a Hankel matrix. The idea is to eliminate the states requiring a large amount of energy to be reached, or a large amount of energy to be observed, as both correspond to small eigenvalues [9]. Grammians are introduced since they can be used to quantify these amounts of energy. The reachability grammian quantifies the energy needed to bring a state to a chosen value, whereas the observability grammian quantifies the energy provided by an observed state [10]. The value of these grammians obviously depends on the basis on which they are calculated. In the case of a stable system, a basis in the state space exists in which states that are difficult to reach are also difficult to observe. Normally, the Hankel singular values decrease rapidly. The balanced truncation aims at truncating the modes that are not reachable and observable. They correspond to the smallest Hankel singular values. The singular value decomposition is well-conditioned, stable and can always work, but can be expensive to compute. It solves high-dimensional Lyapunov equations [11]; the storage required is of the order  $O(n^2)$ , while the number of operations is of the order  $O(n^3)$ . Many balancing methods exist, such as stochastic balancing, bounded real balancing, positive real balancing [12]. The frequency weighted balancing [13], can be useful if a good approximation is needed only in a specific frequency range. However, the reduced model is not necessarily stable if both input and output are weighted. These frequency weighted balancing methods have undertaken many improvements: the most recent one guarantees stability and yields to a simple error bound [14]. Based on Markov parameters, the Padé approximation (moment matching method) [15] has then been improved by Arnoldi and Lanczos [4] and is particularly recommended in the case of high dimension systems.

The reduced order model developed in this paper falls into the second category of approach and is described in the following section.

## II. Reduced order model

For given flow conditions, the frequency response of the integrated aerodynamic coefficients obtained with a CFD code is directly related to the frequency of the pitching motion. It is therefore appropriate to build a reduced order model of the frequency response in the frequency domain instead of performing a classical reduction in the time domain. After solving the system and transforming back into the continuous time space, it is possible to reconstruct any motion in the time domain. The conversion between continuous and discrete time or frequency spaces is achieved using a bilinear transform, as it is a bijective function from  $[0, \pi]$  to  $[0, \infty]$ .

The developed method gives accurate results when applied to a pitching airfoil in the transonic range, with no shock-induced separation. It uses the Eigensystem Realization Algorithm [16], based on the singular value decomposition to keep the dominant modes of the frequency response. The method proposed enables a model based on experimental data to be built without knowing the system matrices.

As it needs equispaced input data in the discrete frequency domain, the choice of the sampling spacing is a key element.

The equispaced discrete frequencies are defined by

$$\hat{\omega}_d(k) = \frac{j\pi}{N}, j \in [0, N] \quad (1)$$

where  $j$  is an integer between 0 and  $\pi$ , and  $\hat{\omega}_d(k)$  is the corresponding discrete frequency.

The relationship to continuous frequencies as a result of the linking bilinear transformation is controlled by the sampling time parameter  $T$  via

$$\omega(k) = \frac{2}{T} \tan \frac{\hat{\omega}_d(k)}{2} \quad (2)$$

$T$  has to be chosen such that the continuous reduced frequencies are in the range of interest for the model input. In aerodynamics it corresponds to continuous reduced frequencies mostly in the interval  $[0.01, 10]$ . As the discrete frequency response is real, the discrete frequency domain between  $[0, \pi]$  can be extended to  $[\pi, 2\pi]$  using the conjugate of the impulse response coefficients  $G_d$ :

$$G_d(k + N) = G_d^*(N - k) \quad (3)$$

A singular value decomposition [17] of the Hankel matrix defined using the  $2N$ -points inverse discrete Fourier transform (IDFT) is performed. The model reduction is performed by keeping the largest singular values.

A discrete-time linear and stable MIMO model of  $n$ -th order, with  $r$ -input and  $m$ -output channels can be described using the following state space representation

$$\begin{aligned} \mathbf{x}(k+1) &= A_d \mathbf{x}(k) + B_d \mathbf{u}(k) \\ \mathbf{y}(k) &= C_d \mathbf{x}(k) + D_d \mathbf{u}(k) \end{aligned} \quad (4)$$

$\mathbf{x}(t) \in \mathbb{R}^n$  represents the vector of different degrees of freedom (called state vector in control theory). It contains for example the unknown physical variables, such as velocity, pressure, density.  $\mathbf{y}(t) \in \mathbb{R}^p$  and  $\mathbf{u}(t) \in \mathbb{R}^m$  respectively represent the vector of the outputs of interest of the system, and the vector of inputs.

Another convenient notation is also used for a discrete-time model:

$$G_d : \begin{pmatrix} A_d & B_d \\ C_d & D_d \end{pmatrix} \quad (5)$$

As far as a continuous-time is concerned, the matrices are written in this paper under the form

$$G : \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad (6)$$

The reduced matrices  $\hat{A}_r$  and  $\hat{C}_r$  are calculated [18]. G can be written as

$$\widehat{G}_d = \hat{C}_d(zI - \hat{A}_d)^{-1}\hat{B}_d + \hat{D}_d, z \in \mathbb{C} \quad (7)$$

The calculation of  $\hat{B}_r$  and  $\hat{D}_r$  is achieved by decomposing G in its real and imaginary parts.

Importantly the method provides a model reduction which is approximately balanced in the discrete space. Furthermore as the bilinear transform below is used to map to the continuous domain, the final continuous time model is also approximately balanced.

G can be transformed in a discrete-time system

$$\hat{G}_d(z) = \hat{D}_d + \hat{C}_d(zI - \hat{A}_d)^{-1}\hat{B}_d \quad (8)$$

using the bilinear transformation [19]:

$$\hat{A} = \frac{2}{T} (I + \hat{A}_d)^{-1} (\hat{A}_d - I) \quad (9)$$

$$\hat{B} = \frac{2}{\sqrt{T}} (I + \hat{A}_d) \hat{B}_d \quad (10)$$

$$\hat{C} = \frac{2}{\sqrt{T}} \hat{C}_d (I + \hat{A}_d)^{-1} \quad (11)$$

$$\hat{D} = \hat{D}_d - \hat{C}_d (I + \hat{A}_d)^{-1} \hat{B}_d \quad (12)$$

### III. Viscous aircraft, numerical model and solver

#### A. Case of interest

The reduced order model technique is being applied to a viscous case, using the Nasa Common Research model [20] (Figure 1) .

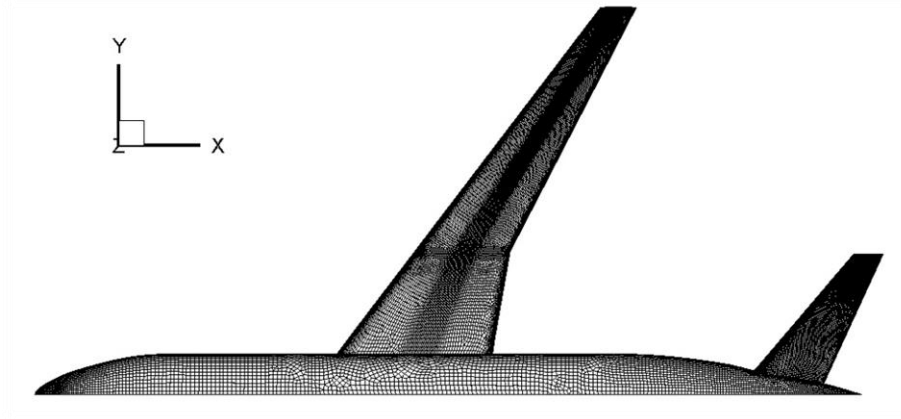


Figure 1: Nasa Common Research model, mesh

The model has to be tested in the transonic regime. Therefore the freestream conditions are :

$$\begin{aligned} \text{Mach} &= 0.86 \\ \text{Reference temperature} &= 228.724 \text{ K} \\ \text{Reference density} &= 0.4588 \\ \alpha_m &= 2 \text{ degrees} \end{aligned}$$

The calculations are processed using 5 multigrid cycles, with the first 50 iterations in single grid. That is the reason why the residual show this evolution (Figure 2). Calculations are achieved in parallel on 16 processors, using central discretization, matrix dissipation. The reference point for the calculation of the moments is at middle of the fuselage in the x and z direction, with  $y=0$ . Steady and unsteady calculations have been performed and show a very good convergence.

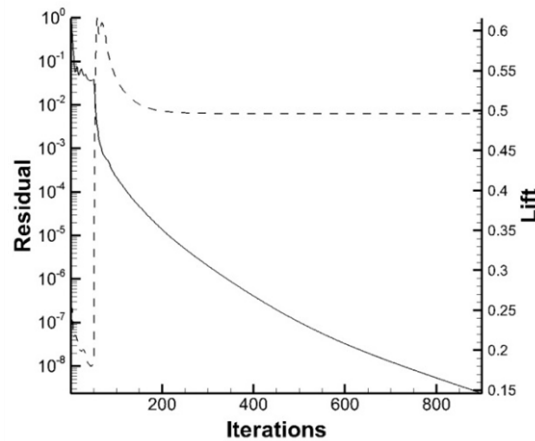


Figure 2 : Steady calculation, residuals.  $Ma=0.86$ ,  $\alpha=2$  degrees

The surface values of  $y^+$  and of the pressure coefficient are represented for the steady case (Figure 3). TAU [21] enables a hybrid-Re treatment of turbulent viscous walls. It enables high values of  $y^+$  to be used (up to  $y^+=50$ ) for transonic flight conditions. The pressure coefficient shows a standard evolution on the surface for such conditions.

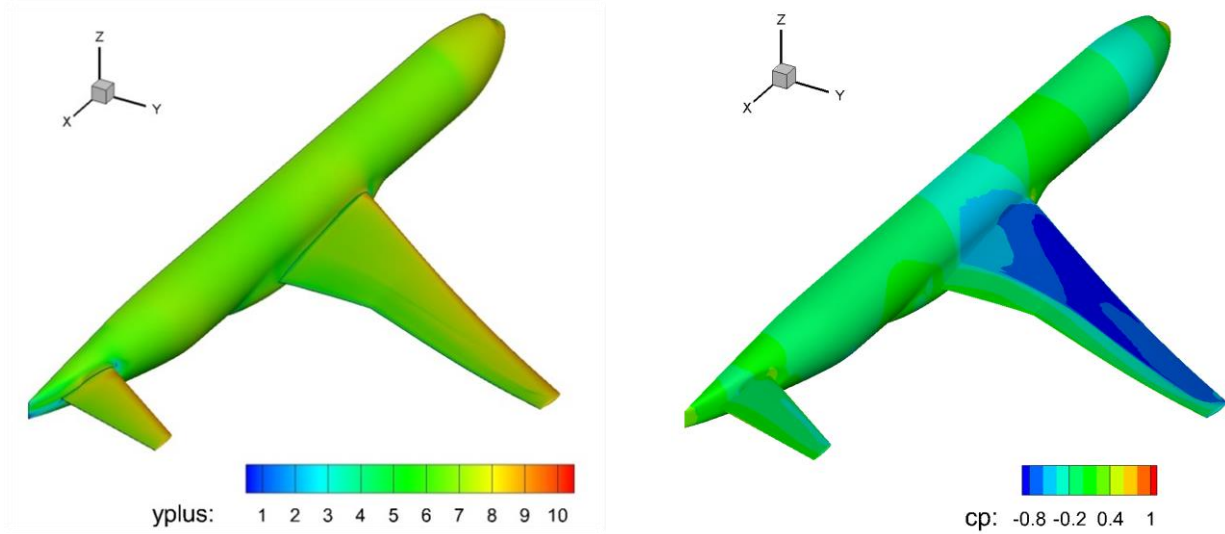


Figure 3 : Steady calculation, surface values of  $y^+$  and pressure coefficient

## B. Unsteady aerodynamics

The motion is sinusoidal and described by the following equation:

$$\alpha = \alpha_m + \alpha_0 \cdot \sin(\omega \cdot t) \quad (13)$$

where  $\alpha_m$  is the mean angle of attack,  $\alpha_0$  the amplitude and  $\omega$  the frequency of the motion. Let  $U_\infty$  be the freestream velocity and  $c$  the airfoil chord, the reduced frequency is defined such as

$$k = \frac{\omega \cdot c}{U_\infty} \quad (14)$$

## C. Linearized frequency domain solver

In addition to the classical time domain computation, as the amplitude of the motion is small and the motion periodic, it is possible to use the linearized frequency domain solver [22].

An unsteady governing equation of the fluid motion discretised in space can be written as

$$\frac{d\mathbf{u}}{dt} + R(\mathbf{u}, \mathbf{x}, \dot{\mathbf{x}}) = 0 \quad (15)$$

where  $R$  is the residual, written as a function of the flow solution  $\mathbf{u}$ , the grid coordinates  $\mathbf{x}$  and the grid velocities  $\dot{\mathbf{x}}$ . Under the assumption of a small amplitude of the unsteady perturbations, the RANS equation can be linearized around the steady state, i.e. it is seen as the superposition of the steady state mean and of the perturbation.

$$\mathbf{u}(t) = \bar{\mathbf{u}} + \tilde{\mathbf{u}}(t), \quad \|\tilde{\mathbf{u}}\| \ll \|\bar{\mathbf{u}}\| \quad (16)$$

$$\mathbf{x}(t) = \bar{\mathbf{x}} + \tilde{\mathbf{x}}(t), \quad \|\tilde{\mathbf{x}}\| \ll \|\bar{\mathbf{x}}\| \quad (17)$$

Since the perturbation is assumed to be periodic, it can be transformed in the frequency domain and expressed as

$$\tilde{x}_k(t) = \sum_k \text{Re}(\hat{x}_k e^{jk\omega t}) \quad (18)$$

where  $\hat{x}_k$  are the complex Fourier coefficients of the motion,  $\omega$  the frequency,  $k$  the mode and  $j$  complex number such as

$$j = \sqrt{-1} \quad (19)$$

After replacing the linearized values of  $u(t)$  and  $x(t)$  in (15), the following system is obtained

$$A\mathbf{x} = \mathbf{b} \quad \text{where} \quad A = \begin{pmatrix} \frac{\partial R}{\partial u} & -\omega I \\ \omega I & \frac{\partial R}{\partial u} \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} \frac{\partial R}{\partial x} & -\omega \frac{\partial R}{\partial \dot{x}} \\ \omega \frac{\partial R}{\partial \dot{x}} & \frac{\partial R}{\partial x} \end{pmatrix} \begin{pmatrix} \tilde{x}_{Re} \\ \tilde{x}_{Im} \end{pmatrix} \quad (20)$$

The Jacobian  $\partial R/\partial u$  is calculated analytically in TAU, but the right hand term is evaluated by using central finite differences.

The harmonics of the aerodynamic coefficients of interest can be obtained using either the linearized frequency solver or a standard time domain simulation. In the case of the time domain simulation, the harmonics are calculated by TAU at the end of each period of the pitching motion. However, it is necessary to wait a few periods so that the values of the harmonics are converged.

Both methods output the magnitude and phase of the aerodynamic coefficients. A comparison is made between the results given by the LFD solver and the one given by a standard time domain simulation. As far as LFD solver calculations are concerned, results have been obtained for 64 different reduced frequencies (Figure 4)

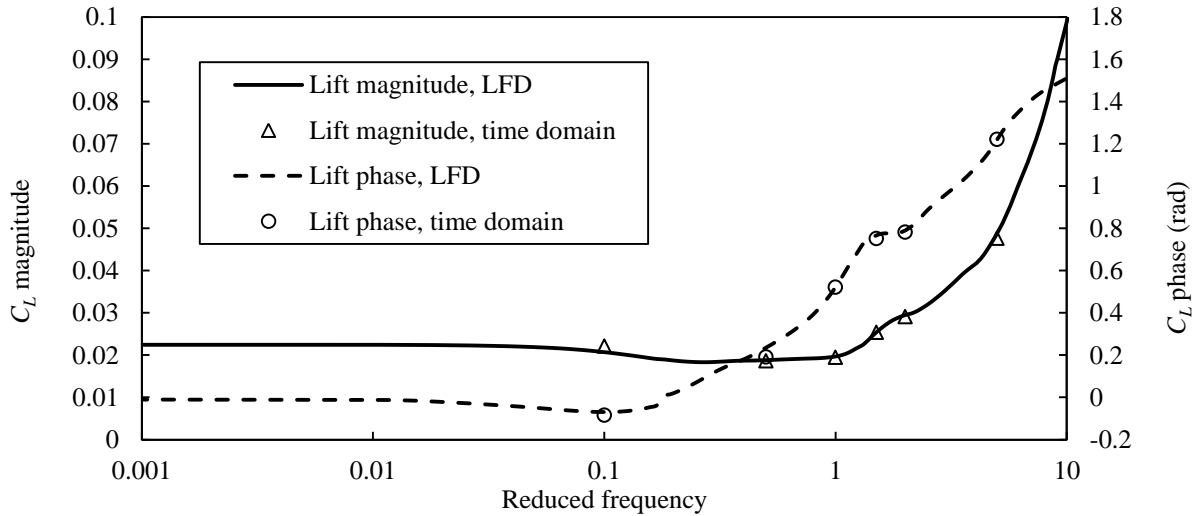


Figure 4: LFD vs time domain simulation, lift magnitude and phase

Since this calculation is really expensive, only few values are given for the time domain results. However, both methods give really similar results for the lift magnitude and phase.



The linearized frequency domain gives satisfactory results when looking at the pitching moment (Figure 5).

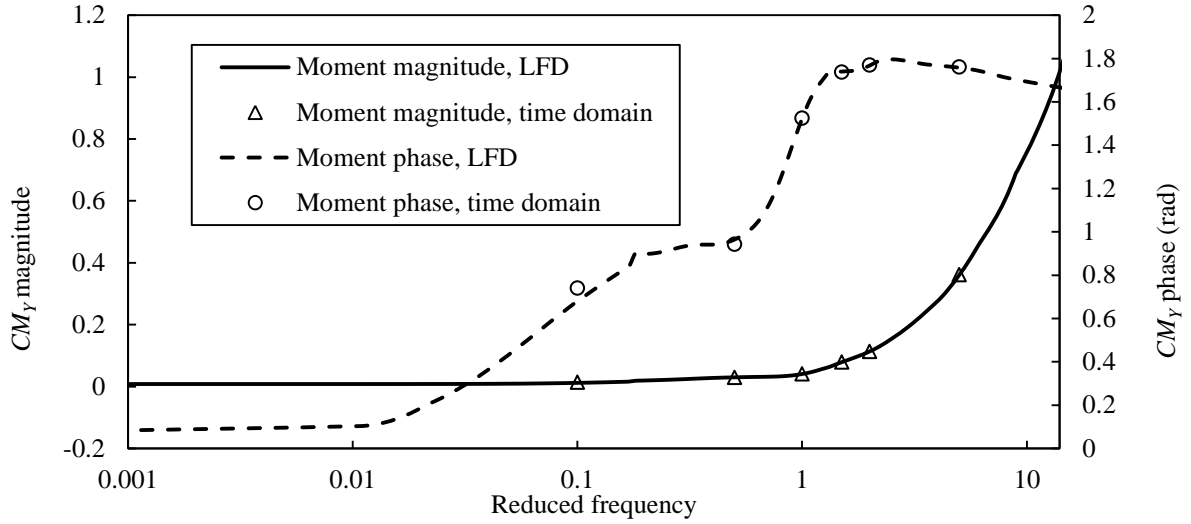


Figure 5: LFD vs time domain simulation, lift and pitching moment

As the linear frequency domain solver gives fast and accurate results, it will be used to give inputs to test the reduced order model.

#### IV. Building a reduced order model

##### A. Slicing the surface mesh

To effectively model the aerodynamic coefficients on the plane, the surface mesh has been sliced in parts (Figure 4). Three different reduced order models are to be built, one for the fuselage, slicing it in different strips along the x axis, a second one to model the forces and moments on the horizontal tail plane. Finally, the wing is sliced in the y direction.

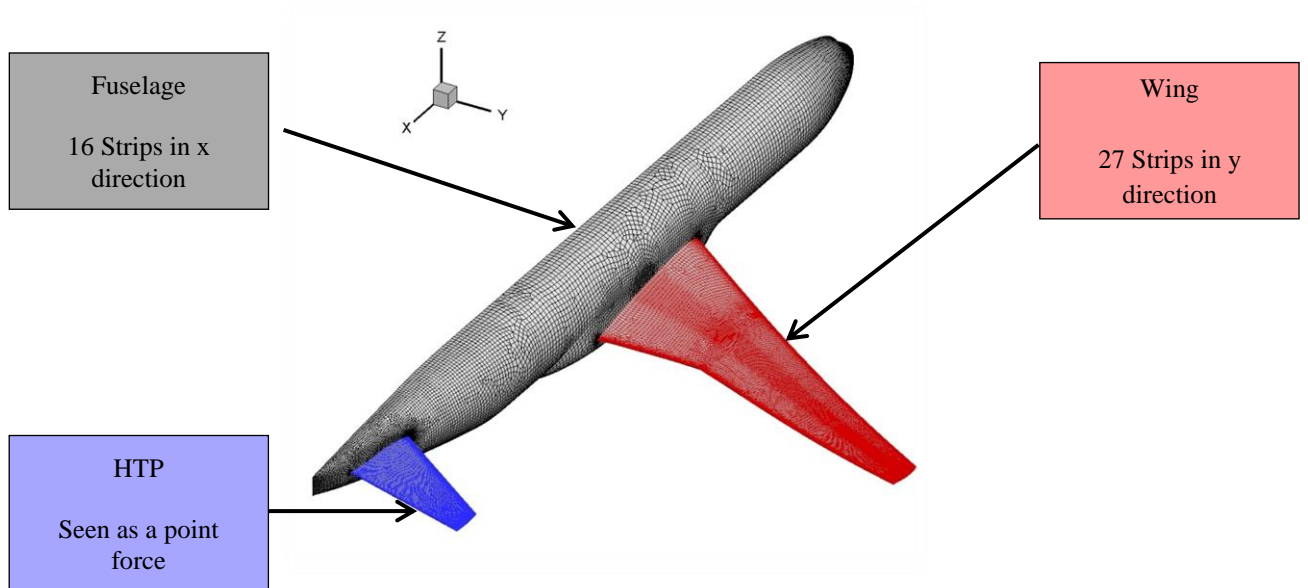


Figure 6: Strip approach

A LFD calculation is launched for each reduced frequency wanted. TAU outputs a solution file containing the position of the nodes of the surface mesh, with the real and imaginary parts of  $C_p$  and  $C_f$  for each node. A script developed by Chris Wales at the University of Bristol enables the user to choose the number of strips he wants, and their position.

For each strip, it extracts from the solution the position of the nodes, before sorting the triangular or quadrilateral elements depending on being contained entirely in this strip or being across its boundary. It uses the TAU solution file to extract the pressure coefficient (and the friction coefficient if desired) for each element, with a clever approach for the elements on the boundary ; it can then easily calculate the total pressure and moment on each strip. In the case of the LFD solver, the outputs given by TAU on the the surface are  $\text{Re}(C_p)$  and  $\text{Im}(C_p)$ , giving  $\text{Re}(F_Z)$ ,  $\text{Im}(F_Z)$ ,  $\text{Re}(M_Y)$  and  $\text{Im}(M_Y)$ .

The magnitude and phase of  $F_Z$  can then be calculated:

$$\text{abs}(F_Z) = \sqrt{\text{Re}(F_Z)^2 + \text{Im}(F_Z)^2} \quad (21)$$

$$\text{arg}(F_Z) = \arctan\left(\frac{\text{Im}(F_Z)}{\text{Re}(F_Z)}\right) \quad (22)$$

The same approach being valid for the pitching moment. For each frequency, the script detailed above is used to give the real and imaginary parts of the aerodynamic coefficients.

The script returns the harmonics of lift and pitching moment for each strip. It enables to represent the harmonics on each strip using a bar chart. In the case of  $\text{Re}(F_Z)$  for example, the total value along the wing is the sum of the values obtained for each strip.

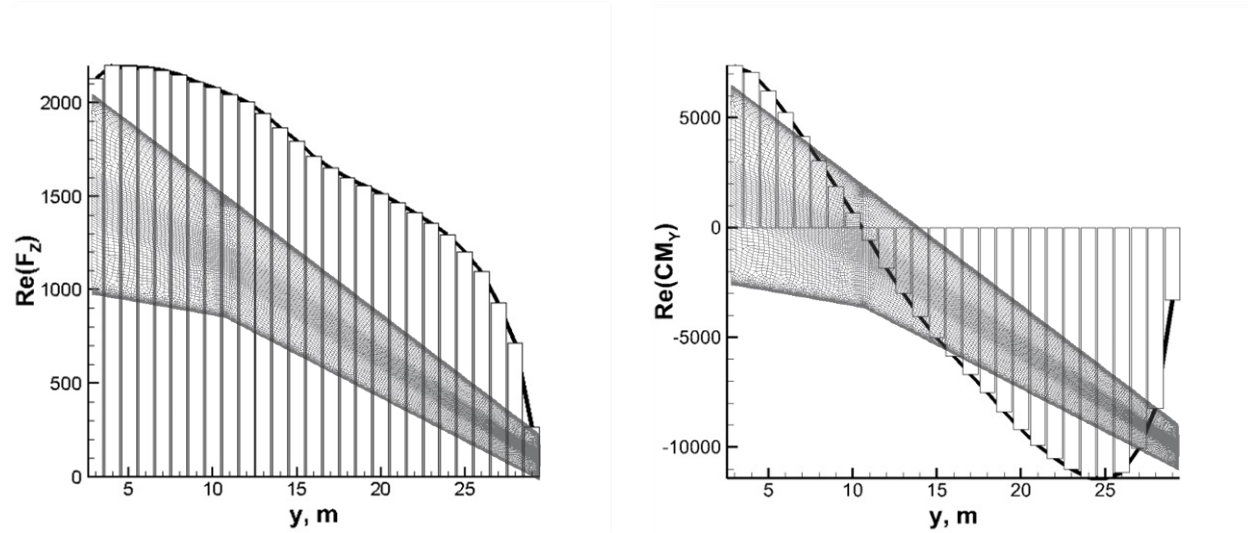


Figure 7: Real part of the vertical force and pitching moment along the wing,  $k=0.2$

This distribution of  $\text{Re}(F_Z)$  is expected for the wing, with  $\text{Re}(F_Z)$  decreasing with the wingspan. As far as the moment along the wing is concerned, its real part reaches a minimum around  $y=24\text{m}$ , where the shock is quite strong.

The same study can be done for the fuselage (Figure 8). Whereas the data for the moment is harder to analyze without further studies,  $\text{Re}(F_z)$  shows a peak where the wing is (between  $x=20$  and  $x=40\text{m}$ ).

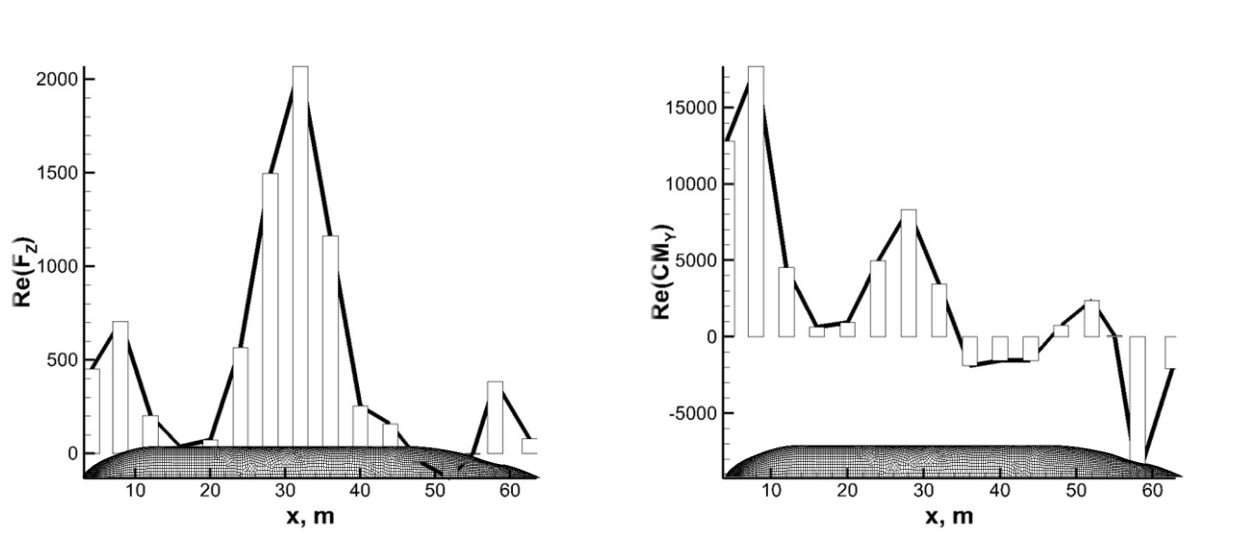


Figure 8: Real part of the vertical force and pitching moment along the fuselage

## B. Building the reduced order models : process

The reference set of data consists of 64 LFD calculations at different frequencies. Due to the choice of the method used to build the ROM, the mapping of these frequencies is driven by the spacing of the bilinear transform. As written in (2), the data has to be equispaced in discrete time. The spacing used is  $T=0.02$ , as gave the best results for a wing [23].

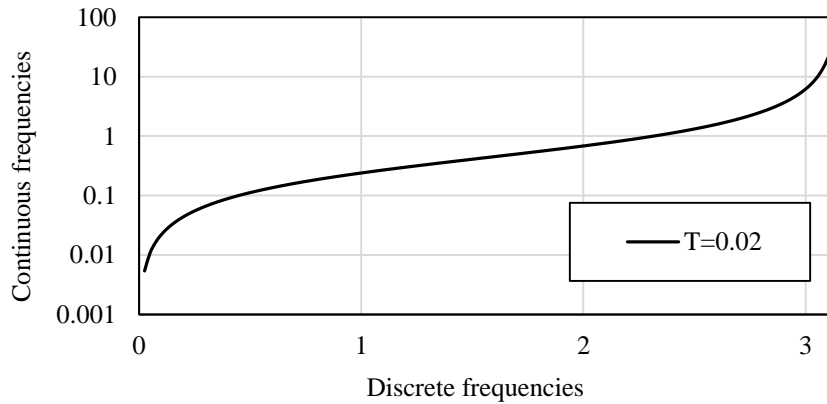


Figure 9: Continuous and discrete frequencies

The model can be used to reconstruct the frequency response  $G_r$  on a chosen number  $N_r$  of equispaced discrete frequencies corresponding to the same number of continuous frequencies.

The process is represented on Figure 10.

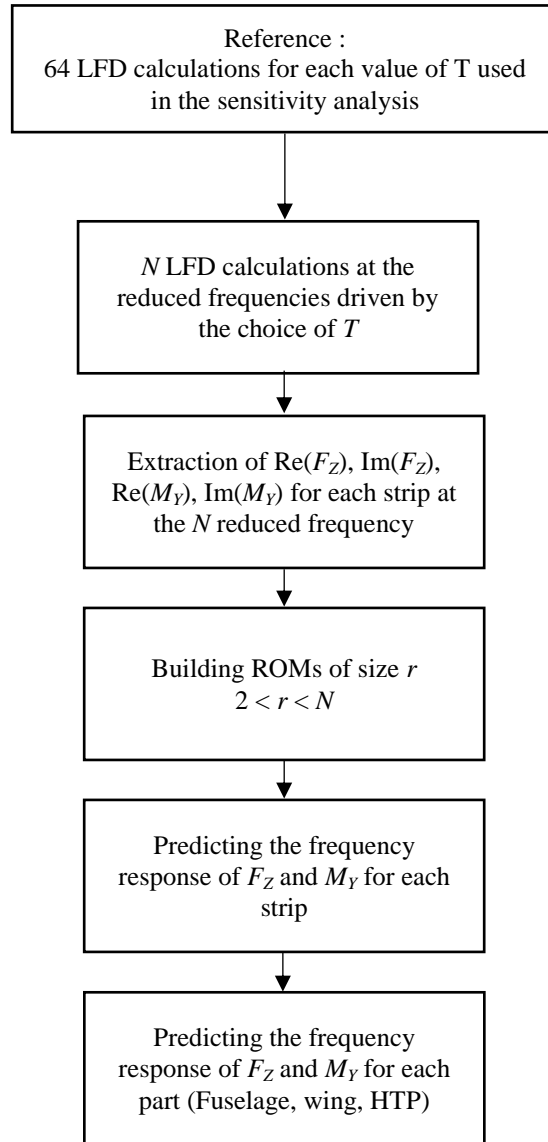


Figure 10: Description of the process

## V. Results

### A. Total forces and moments on the fuselage

For each strip of the fuselage, reduced order models of various sizes are created. The prediction of these ROMs can be compared to the reference, which is the value given by the LFD solver. For a better understanding, an example is given of the predictions of the harmonics of the vertical force, for a strip in the middle of the fuselage. Predictions of ROMs of sizes  $r=3$ ,  $r=9$  and  $r=13$  can be compared to the reference values.

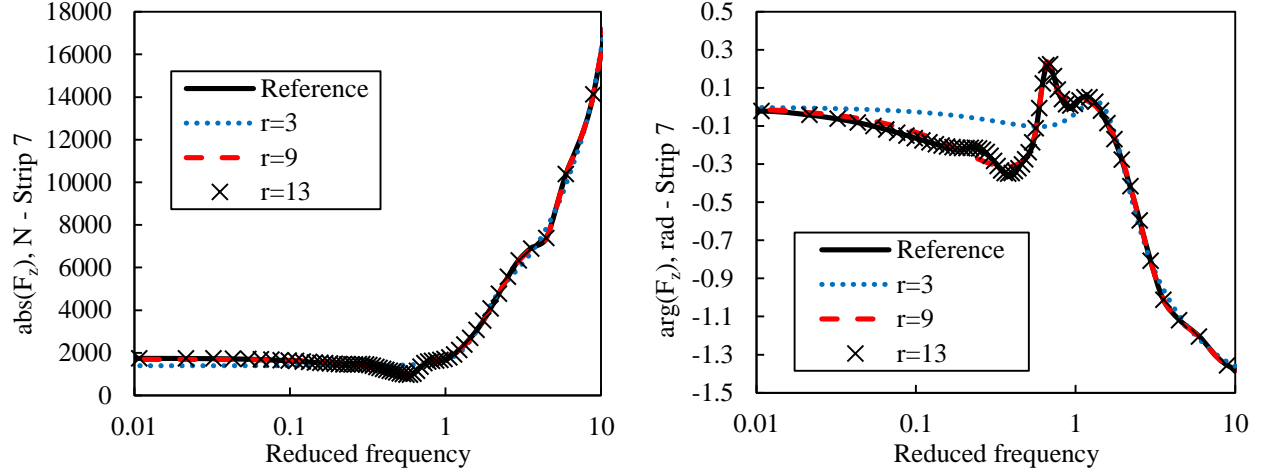


Figure 11: reconstruction of  $F_z$  magnitude and phase, Strip 7 - Fuselage

As far as the magnitude of the vertical force on Strip 7 is concerned, all ROMs are accurate apart from the one of size  $r=3$ . It underestimates the magnitude for low frequencies. The phase is more challenging to model since there are more dynamics to catch, and from  $r=9$ , the ROM prediction is really good.

Having these predictions for each strip, it is possible to model the behavior of the whole fuselage in a pitching motion (Figure 12).

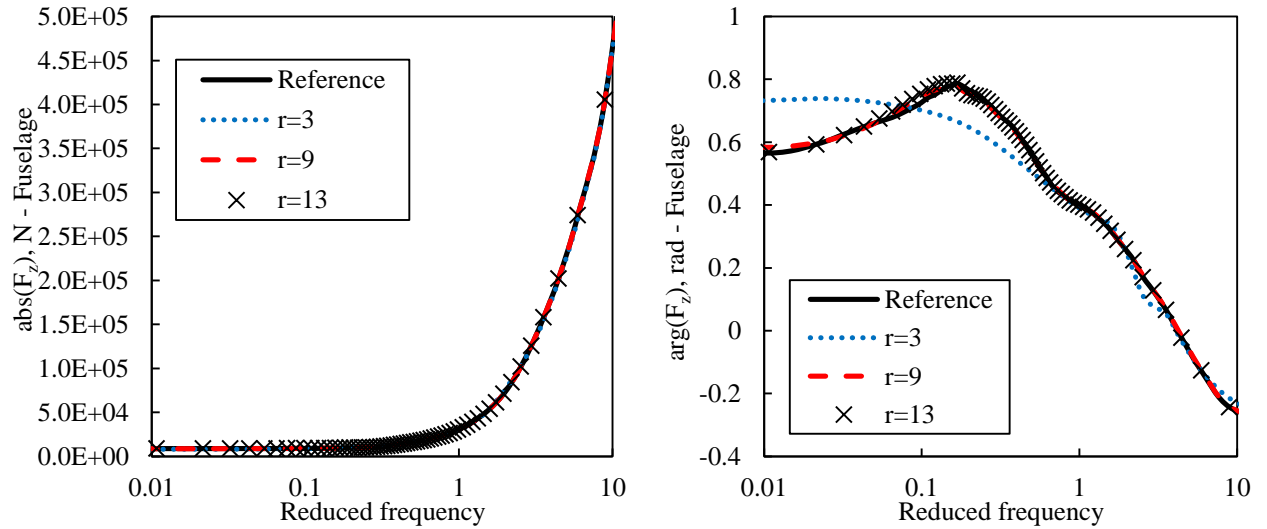


Figure 12: reconstruction of  $F_z$  magnitude and phase, Fuselage

The fuselage is a complex part since the variations in pressure coefficient between the different strips can be really high. However, for both magnitude and phase, small ROMs ( $r=9$ ) give an accurate prediction of the aerodynamic coefficient.

## B. Total forces and moments on the wing

The same technique is used to predict the magnitude and phase of the aerodynamic coefficients on the wing (Figure 13 and .

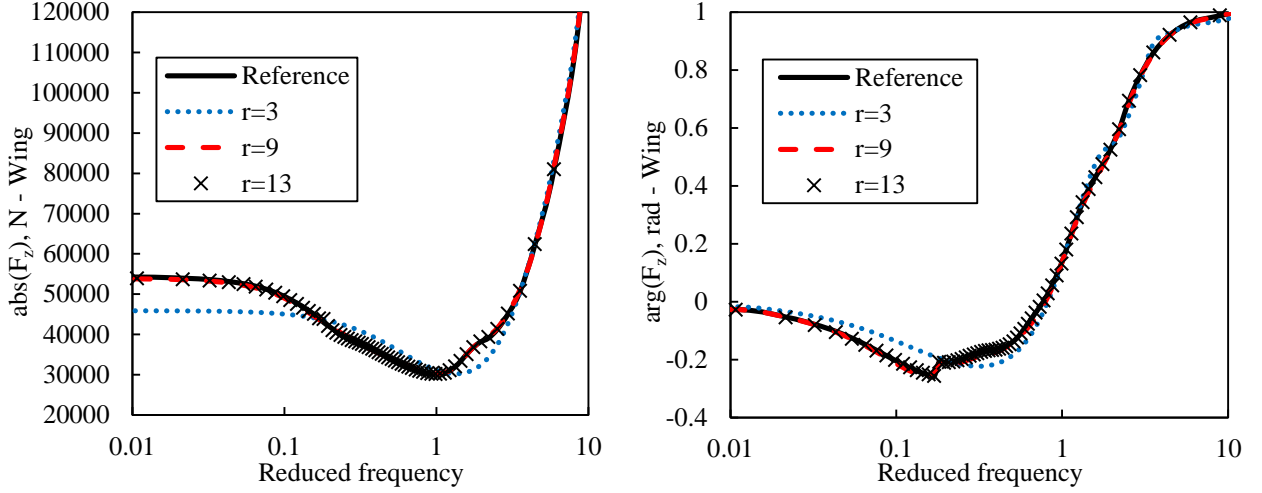


Figure 13: reconstruction of  $F_z$  magnitude and phase

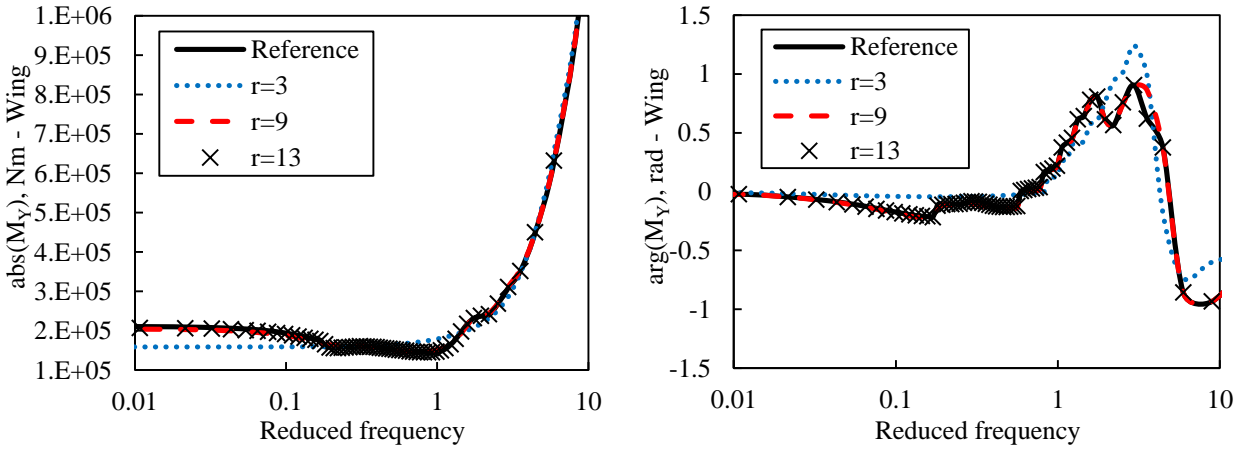


Figure 14: reconstruction of  $F_z$  magnitude and phase

The quality of the prediction is really interesting, especially when looking at the phase of the vertical force and of the pitching moment.

### C. Total forces and moments on the HTP

The horizontal tail plane is not sliced in different parts, so the reduced order models are directly built for the whole part (Figure 15)

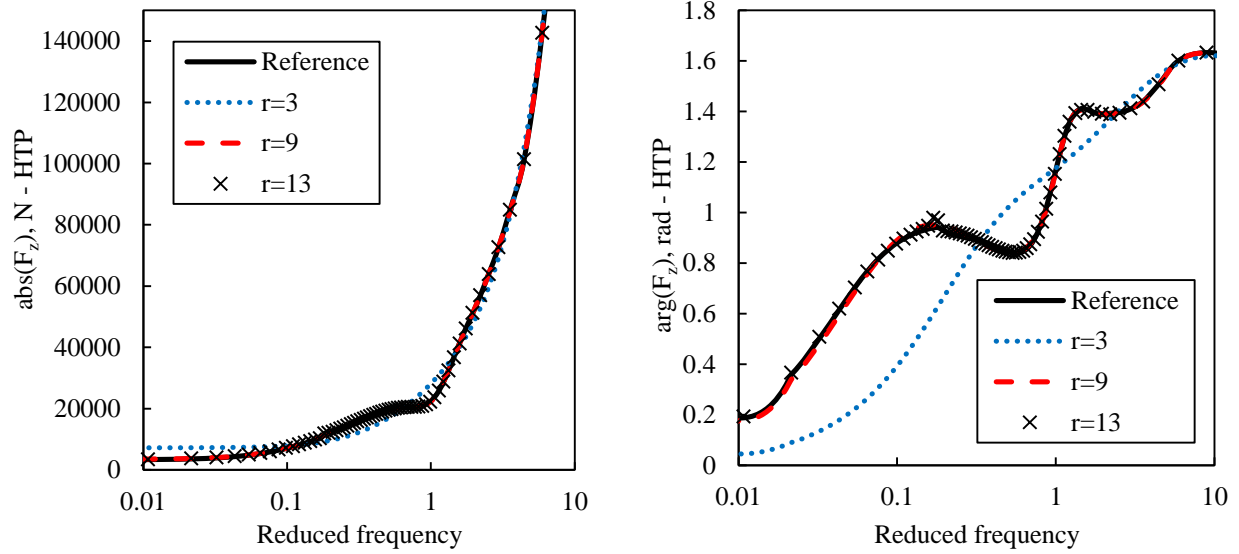


Figure 15: reconstruction of  $F_z$  magnitude and phase

The results obtained for the pitching moment are very close to the one obtained for the vertical force and show a very good accuracy in the prediction. However, the ROM of size  $r=3$  can not catch enough dynamics to model the phase properly.

## VI. Conclusions

A reduced order model in the frequency domain has been built and shows a strong ability to reconstruct the frequency response. Applied to a viscous aircraft model, it enables the prediction of the loads to be done at almost no computational cost. To be efficient, robust and more reliable, the different parts of the aircraft have been sliced in different strips and reduced order models have been built for the lift and pitching moment for each of these strips. The total prediction of the forces and moments is highly accurate when compared to the reference values.

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